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Two Proofs of Cauchy's Theorem.

BY F. FRANKLIN.

The following proofs of Cauchy's theorem that $\int_1^2 w dz$ has the same value for any two paths joining the points 1 and 2, provided that w is a uniform and continuous function of $z (= x + i)$ throughout the area included between those paths, are very simple; and I think they have the merit of more pointedly turning on the fundamental property that dw/dz is independent of the direction of dz than do the proofs usually given.

1°. Let the integral be taken along a certain path, and let the path be slightly deformed; denote the effect of this deformation by δ ; then

$$\begin{aligned}\delta \int w dz &= \int \delta (w dz) = \int \delta w \cdot dz + \int w \cdot \delta (dz) = \int \delta w \cdot dz + \int w d(\delta z) \\ &= [w \delta z]_1^2 + \int (\delta w \cdot dz - \delta z \cdot dw) \\ &= \int (\delta w \cdot dz - \delta z \cdot dw),\end{aligned}$$

since δz is zero at the points 1 and 2. All this is true whether w be a function of z or merely a function of x and y . But if w is a function of z , uniform and continuous in the region in question, $\delta w/\delta z = dw/dz$, or $\delta w \cdot dz - \delta z \cdot dw = 0$; hence the deformation does not affect the integral. This proves the theorem.

2°. Consider an infinitesimal contour, c , containing the point z_0 . At the point z_0 let $w = w_0$, and put $z = z_0 + \zeta$, $w = w_0 + \omega$. Then

$$\begin{aligned}\int_c w dz &= \int_c (w_0 + \omega) d\zeta = w_0 \int_c d\zeta + \int_c \omega d\zeta \\ &= \int_c \omega d\zeta,\end{aligned}$$

since $\int_c d\zeta$ is obviously 0. Now, if w is a function of z (uniform and continuous

in the region considered), ω/ζ is constant, $= A$ say, around the contour by the fundamental property already referred to; hence

$$\int_c \omega dz = A \int_c \zeta d\zeta = \frac{1}{2} A \int_c d(\zeta^2) = 0.$$

Thus the integral taken around an infinitesimal contour vanishes. Hence, by addition, the integral taken around any contour vanishes, and the theorem is proved.

Strictly speaking, what was proved about $\int_c \omega dz$ is not that it is absolutely 0, but that it is an infinitesimal of a higher order than the second; but this is of course sufficient for the purpose.

It may be added that neither of these proofs depends on the fact that $z = x + iy$; they are equally applicable if z is any function of x and y , and w a function of z ; z and w being supposed uniform and continuous in the region concerned.

BALTIMORE, Feb. 19, 1887.